Short handbook of the most useful formulas

Definitions

$X_i(t)$: offered traffic in class *i* in a scheduler

$Y_i^*(t)$ or $\widehat{Y}_i(t)$: available service for class *i* in a scheduler

E(): average value operator

V(): variance operator

Alpha function

$$\alpha(t) = -\frac{E(X_i) - E(\widehat{Y}_i(t+d))}{\sqrt{V(X_i(t)) + V(\widehat{Y}_i(t+d))}}$$

Delay tail

Given the absolute minimum of $\alpha(t)$, referred to as α_{min} :

$$Pr(D \geqslant d) = e^{\frac{-\alpha_{min}^2}{2}}$$

FIFO scheduler

$$\widehat{Y}_i(t) = Ct$$

$$E(\widehat{Y}_i(t)) = Ct$$

$$V(\widehat{Y}_i(t)) = 0$$

Stric priority scheduler

$$\widehat{Y}_i(t) = Ct - \sum_{j=1}^{i-1} X_i(t)$$

$$E(\widehat{Y}_i(t)) = Ct - \sum_{j=1}^{i-1} E(X_i(t))$$

$$V(\widehat{Y}_i(t)) = \sum_{j=1}^{i-1} V(X_i(t))$$

GPS scheduler

$$\widehat{Y}_i(t) = w_i Ct + \sum_{j \neq i} \frac{w_i}{1 - w_j}(w_j Ct - X_j(t))$$

$$E(\widehat{Y}_i(t)) = w_i Ct + \sum_{j \neq i} \frac{w_i}{1 - w_j} E(w_j Ct - X_j(t))$$

$$V(\widehat{Y}_i(t)) = \sum_{j \neq i} \left(\frac{w_i}{1 - w_j}\right)^2 V(X_j(t))$$

EDF scheduler

$$\widehat{Y}_i(t) = Ct - \sum_{j \neq i} X_j(t - max(0, \delta_j - \delta_i))$$

$$E(\widehat{Y}_i(t)) = Ct - \sum_{j \neq i} E(X_j(t - max(0, \delta_j - \delta_i)))$$

$$V(\widehat{Y}_i(t)) = \sum_{j \neq i} V(X_j(t - max(0, \delta_j - \delta_i)))$$

FIFO scheduler with SRD traffic

A FIFO scheduler with capacity *C*, offered *N* SRD traffic flows, where each traffic flow has parameters *r*, *b*, and *H*=1/2. The requested SLA is (*d*,*p*).

Delay tail

$$Pr(D>d)=\exp(-2\frac{C(C-Nr)}{Nrb}d)$$

Average delay

$$E(D)=\frac{Nrb}{2C(C-Nr)}$$

Admission control

$$N\leqslant\frac{2C^2d}{2Cdr-rb\ln(p)}$$

Resource provisioning

$$C\geqslant\frac{Nr}{2}+\sqrt{(\frac{Nr}{2})^2-\frac{Nrb\ln(p)}{2d}}$$

FIFO scheduler with LRD traffic

A FIFO scheduler with capacity *C*, offered *N* SRD traffic flows, where each traffic flow has parameters *r*, *b*, and *H*. The requested SLA is (*d*,*p*).

Delay tail

$$Pr(D>d)=\exp(\frac{-1}{2Nrb}\frac{1}{H^{2H}(1-H)^{2-2H}}C^{2-2H}(C-Nr)^{2H}d^{2-2H})$$

Strict Priority (SP) scheduler with SRD traffic

A SP scheduler with capacity *C*, offered N_i SRD traffic flows in class *i*, where each traffic flow in class *i* has parameters r_i, b_i, and $H_i=1/2$. The requested SLA for class *i* is (d_i,p_i).

Delay tail

Define

$$A_i=\sum_{j=1}^{i}N_jr_j$$

(note that $A_0=0$)

$$B_i=\sum_{j=1}^{i}N_jr_jb_j$$

(note that $B_0=0$)

$$Pr(D_i>d_i)=\exp(-2\frac{C-A_i}{B_i^2}[(C-A_{i-1})B_i-(C-A_i)B_{i-1}]d)$$

Modification of the variance of a traffic flow when crossing a scheduler

$$V(X_{i,out}(t))=max(V(X_i(t)),\hat{Y}_i(t))$$

Probability density of delay from delay tail

$$f_D(t) = \frac{-d}{dt} Pr(D > t)$$

Probability density of end-to-end delay

When a traffic flow crosses a series of schedulers, the delay in scheduler i is D_i, and the total delay will be

$$D_{tot} = D_1 + D_2 + \ldots + D_n \quad ,$$

where n is the number of schedulers. The probability density of D_{tot} is

$$f_{D_{tot}}(t) = f_{D_1}(t) \otimes f_{D_2}(t) \otimes \ldots \otimes f_{D_n}(t)$$

where $\otimes$ denotes the convolution operation.

If D_1, …, D_n, are identical exponential variables, that is

$$\forall i : f_{D_i}(t) = k\, e^{-kt}$$

then the probability density of the end-to-end delay is an Erlang

$$f_{D_{tot}}(t) = \frac{k^n t^{n-1}}{(n-1)!} e^{-kt}$$

If D_1, …, D_n, are exponential variables with different parameter k, that is

$$\forall i, j \neq i : f_{D_i}(t) = k_i\, e^{-k_i t} ; f_{D_j}(t) = k_j\, e^{-k_j t}, k_i \neq k_j$$

then the probability density of end-to-end delay is an hyperexponential. In the case of n=2:

$$f_{D_{tot}}(t) = \frac{k_1 k_2}{k_2 - k_1} \left(e^{-k_1 t} - e^{-k_2 t} \right)$$

SRD model of VoIP traffic, codec with Voice Activity Detection

$$E(X(t)) = \frac{\lambda}{\lambda + \mu} P\, t$$

$$V(X(t)) = 2 \frac{\lambda \mu}{(\lambda + \mu)^3} P^2\, t$$

where

$$\lambda = 1.538\, s^{-1}$$

$$\mu = 2.8571\, s^{-1}$$