The strict priority scheduler

- The strict priority scheduler is very simple and efficient
- In the following, we will carry out the analysis of this scheduler to determine the formulas for performance analysis, resource allocation and admission control
- The service envelope of service priority *i*, in the strict priority scheduler, is

$$S_i(t) = \max\left(0, Ct - \sum_{j=1}^{i-1} X_j(t)\right)$$

- Where *C* is the line capacity
- In fact, service class *i* is interfered only by service classes with a better service priority, that is, classes from 1 to *i*-1

The strict priority scheduler

- Thus

$$\alpha_i(t) = -\frac{E\left(X_i(t) - \max\left(0, C(t+d_i) - \sum_{j=1}^{i-1} X_j(t+d_i)\right)\right)}{\sqrt{\operatorname{var}\left(X(t) - \max\left(0, C(t+d_i) - \sum_{j=1}^{i-1} X_j(t+d_i)\right)\right)}}$$

- The specific form of $\alpha_i(t)$ depends on the average values and variances of traffic flows from 1 to i-1

The strict priority scheduler

- When all traffic flows admit a linear variance envelope, a closed-form analysis is possible
- In this case, the average value of $S_i(t)$ is approximated as

$$E\left(S_i\left(t\right)\right) = Ct - \sum_{j=1}^{i-1} N_j r_j t$$

- And the variance of $S_i(t)$ is approximated as

$$\mathrm{var}\left(S_i\left(t\right)\right) = \sum_{j=1}^{i-1} N_j r_j b_j t$$

- Thus,

$$\alpha_i\left(t\right) = \frac{-N_i r_i t + C\left(t+d_i\right) - \sum_{j=1}^{i-1} N_j r_j\left(t+d_i\right)}{\sqrt{N_i r_i b_i t + \sum_{j=1}^{i-1} N_j r_j b_j\left(t+d_i\right)}}$$

The strict priority scheduler

- The absolute minimum of $\alpha_i(t)$ is

$$\alpha_{i,\min} = 2\sqrt{\frac{(C-A_i)\left((C-A_{i-1})B_i-(C-A_i)B_{i-1}\right)}{B_i^2}d_i}$$

- where

$$A_i = \sum_{j=1}^{i} N_j r_j \qquad B_i = \sum_{j=1}^{i} N_j r_j b_j$$

- From which we can calculate the closed-form formulas for
  - Delay distribution
  - Capacity planning
  - Admission control

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The strict priority scheduler: delay distribution with linear-bounded variance traffic

- Delay distribution

$$\Pr\left(D_i > d_i\right) \approx \exp\left(-2\frac{C-A_i}{B_i^2}\left(\left(C-A_{i-1}\right)B_i - \left(C-A_i\right)B_{i-1}\right)d_i\right)$$

- Average delay

$$E\left(D_i\right) \approx \frac{B_i^2}{2\left(C-A_i\right)\left(\left(C-A_{i-1}\right)B_i - \left(C-A_i\right)B_{i-1}\right)}$$

The strict priority scheduler: capacity planning with linear-bounded variance traffic

- Given a strict priority scheduler with n service classes, where the traffic of class i, $X_i(t)$ has average value $E(X_i(t))=r_i t$ and variance upper bounded by $\text{var}(X_i(t)) \le r_i b_i t$
- Given the statistical delay SLA of class i: (d_i, p_i)
- The line capacity C needed to satisfy concurrently all SLAs is obtained by inverting the delay distribution formula

$$C_i \ge \frac{\left(2A_iB_{i-1} - A_{i-1}B_i - A_iB_i\right)d_i - \sqrt{A_{i-1}^2B_i^2d_i^2 - 2A_{i-1}A_iB_i^2d_i^2 + A_i^2B_i^2d_i^2 + 2B_{i-1}B_i^2d_i\ln\left(p_i\right) - 2B_i^3d_i\ln\left(p_i\right)}}{2\left(B_{i-1} - B_i\right)d_i}$$

- Then:

$$C = \sup_{1 \le i \le n}\left(C_i\right)$$

The strict priority scheduler: capacity planning with linear-bounded variance traffic

- The required capacity depends on the priority level assigned to each service class
- Given *n* service classes, there are *n*! possible permutations for this assignment
- The problem of finding the optimal permutation is NP-complete
- Frequently, the best permutation is that assigning the traffic with the largest average value to priority 1, the second-largest average value to priority 2 and so on
- However, this is not necessarily true in the general case, as also variance has a significant effect

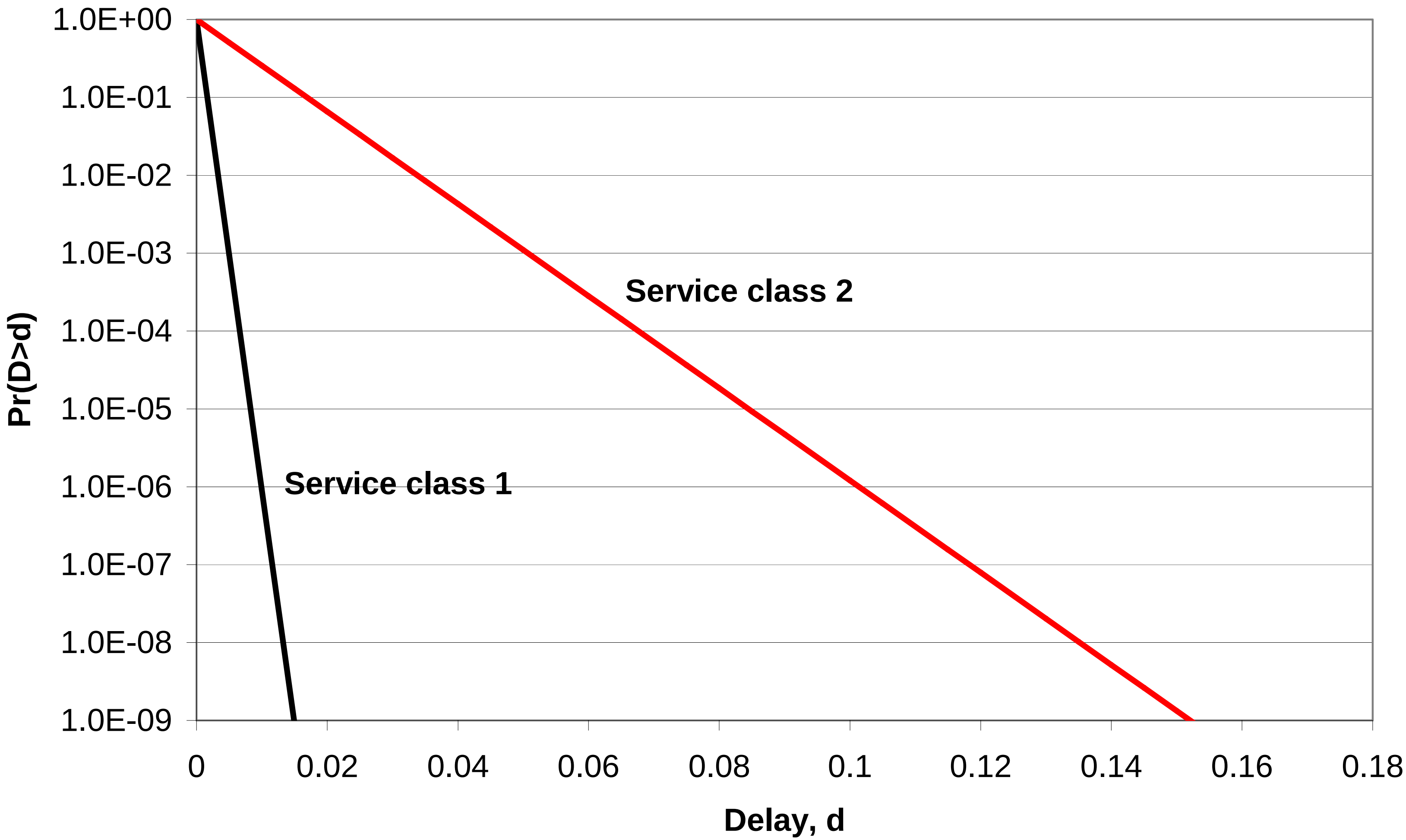
The strict priority scheduler: admission region with linear-bounded variance traffic

- Given a strict priority scheduler with n service classes, where the traffic of class i, $X_i(t)$ has average value $E(X_i(t))=r_i t$ and variance upper bounded by $var(X_i(t)) \leq r_i b_i t$
- Given the statistical delay SLA of class i: (d_i, p_i)
- Given the line capacity C
- The admission region is the set of tuples $(N_1, N_2, ..., N_n)$ for which the SLAs are concurrently fulfilled
- The admission region is calculated by inverting the delay curve
- Algebraic calculations are very complex, but it is possible to find a closed-form expression of the admission region

The strict priority scheduler: an example of calculation of delay curves

- For example, let us consider the case of two service classes, with the following parameters
  - r_1 = 200 kbit/s, b_1 = 9,600 bit
  - r_2 = 200 kbit/s, b_1 = 4,600 bit
  - N_1 = 30 sources
  - N_2 = 10 sources
  - C=1x10^7 bit/s
- The delay curves is this case are calculated with the proposed formula and they are plotted in the next slide

The strict priority scheduler : an example of calculation of delay curves



The strict priority scheduler : an example of calculation of capacity planning

- Continuing with the previous example, let us assume that capacity is unknown and let us add statistical delay SLAs
  - r_1 = 200 kbit/s, b_1 = 9,600 bit
  - r_2 = 200 kbit/s, b_1 = 4,600 bit
  - d_1 = 50 ms, $p_1 = 1 \times 10^{-3}$
  - d_2 = 70 ms, $p_2 = 2 \times 10^{-3}$
  - N_1 = 30 sources
  - N_2 = 10 sources
- The resulting capacity requested to fulfill concurrently all SLAs is:
  - $C_1 = 6.6 \times 10^6$ bit/s
  - $C_2 = 9.35 \times 10^6$ bit/s
- Thus
  - $C = \max(6.6 \times 10^6 \text{ bit/s}, 9.35 \times 10^6 \text{ bit/s}) = 9.35 \times 10^6$ bit/s

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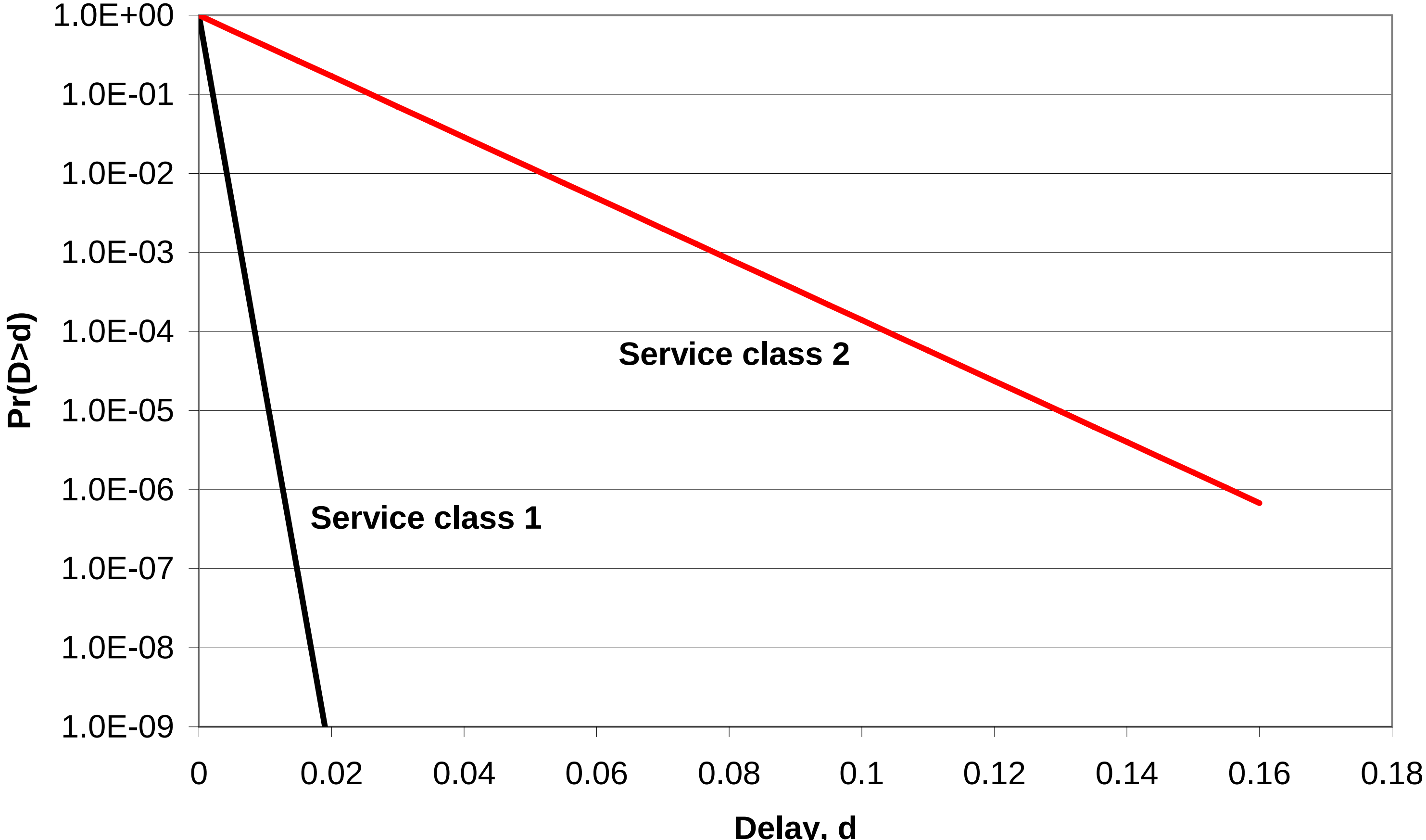
The strict priority scheduler : an example of calculation of capacity planning



- In the scenario of the previous example, the delay curve is that plotted in the figure; the SLA of service class 2 is provided exactly, while the dimensioning for class 1 is large

The strict priority scheduler: admission region with linear-bounded variance traffic

$$\begin{cases} N_i \le \dfrac{\left(\left(\dfrac{B_{i-1}}{b_i}\right)^2 - \left(2C-2A_{i-1}+\dfrac{B_{i-1}}{b_i}\right)^2 - 4B_{i-1}\dfrac{\ln p_i}{d_i}\right) - 4\left(C-A_{i-1}+\dfrac{B_{i-1}}{b_i}\right)\sqrt{\left(C-A_{i-1}\right)^2+2B_{i-1}\dfrac{\ln p_i}{d_i}}}{-4r_i\left(2\left(C-A_{i-1}+\dfrac{B_{i-1}}{b_i}\right)-\dfrac{b_i \ln p_i}{d_i}\right)} & \text{for } 0 \le N_{i-k} \le \min\left(N_{i-k}^*, N_{i-k,\max}\right), \text{and } i>k=1,2,\dots,n \\ N_i = 0 & \text{for } N_{i-k}^* \le N_{i-k} \le N_{i-k,\max}, \text{and } i>k=1,2,\dots,n \end{cases}$$

- where

$$N_{i-k}^* = \frac{\left(Cd_i - b_{i-k}\ln p_i - d_i A_{i-1-k}\right) - \sqrt{-\ln p_i\left(-b_{i-k}^2 \ln p_i + 2d_i b_{i-k}\left(C-A_{i-1-k}\right)+2d_i B_{i-1-k}\right)}}{r_{i-k}d_i}$$

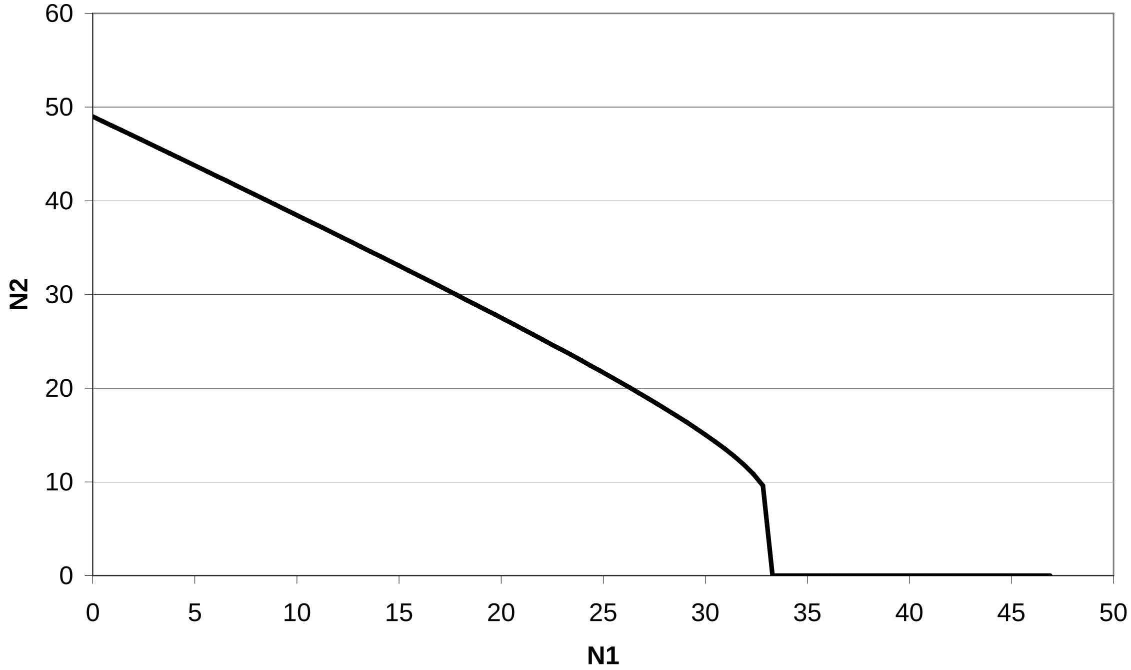
$$N_{i-k,\max} = \frac{2d_{i-k}C^2}{2d_{i-k}r_{i-k}C - r_{i-k}b_{i-k}\ln p_{i-k}}$$

Admission control curves for the SP scheduler

- For example, with the selected parameters, the admission control region is plotted in the figure
- It is interesting to note that for more than 34 sources of class 1, no sources of class 2 can be admitted
- This depends on the specific values of the TCA and SLA parameters
- With other parameters, this phenomenon does not necessarily occur

class 1			class 2		
r1	2.00E+05	bit/s	r2	2.00E+05	bit/s
b1	9.60E+03	bit	b2	4.60E+03	bit/s
r1b1	1.92E+09	bit2/s	r2b2	9.20E+08	bit2/s
d1	0.05		d2	0.07	
p1	1.00E-03		p2	2.00E-03	

C	1.00E+07



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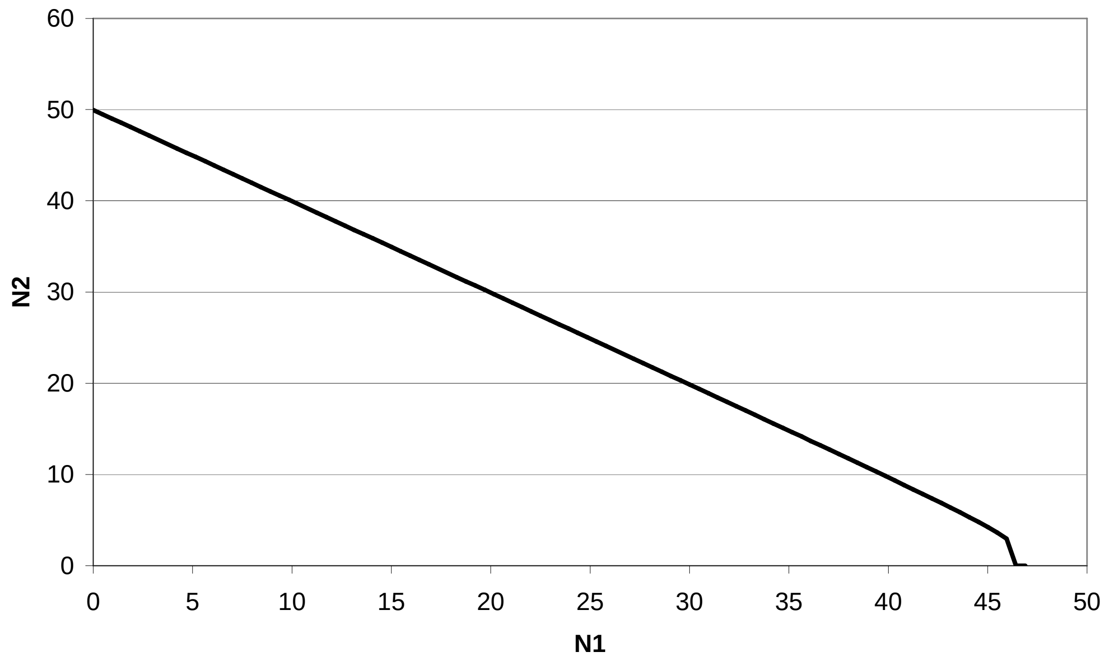
Admission control curves for the SP scheduler

- This is another example, providing a qualitatively different admission region

class 1			class 2		
r1	2.00E+05	bit/s	r2	2.00E+05	bit/s
b1	9.60E+03	bit	b2	4.60E+03	bit/s
r1b1	1.92E+09	bit2/s	r2b2	9.20E+08	bit2/s
d1	0.05		d2	0.8	
p1	1.00E-03		p2	1.00E-01	

C	1.00E+07



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The GPS scheduler

- In a GPS with *n* service classes the statistical service of the class-*i* traffic is given by

$$S_i(t) = w_i C t + \sum_{j \neq i} \frac{w_i}{\sum_{k \neq j} w_k} \max\left(0, w_j C t - X_j(t)\right), \qquad \begin{cases} j = 1, \dots, n \\ k = 1, \dots, n \quad \text{backlogged classes} \end{cases}$$

- Where the *j* index represents non-backlogged classes while the *k* index refers to backlogged classes
- Therefore, the average value of the statistical service envelope available for service class *i* can be evaluated as

$$\alpha_i(t) = -\frac{E\left(X_i(t) - \left(w_i C(t+d_i) + \sum_{j \neq i} \frac{w_i}{\sum_{k \neq j} w_k} \max\left(0, w_j C(t+d_i) - X_j(t+d_i)\right)\right)\right)}{\sqrt{\mathrm{var}\left(X(t) - w_i C(t+d_i) + \sum_{j \neq i} \frac{w_i}{\sum_{k \neq j} w_k} \max\left(0, w_j C(t+d_i) - X_j(t+d_i)\right)\right)}}$$

The GPS scheduler

- With linear-variance bounded traffic,

$$E\left(S_i\left(t\right)\right) \approx w_i Ct + \sum_{j \neq i} \frac{w_i}{\sum_{k \neq j} w_k} \cdot \left(w_j Ct - N_j r_j t\right)$$

$$\mathrm{var}\left(S_i\left(t\right)\right) \approx \sum_{j \neq i} \left(\frac{w_i}{\sum_{k \neq j} w_k}\right)^2 N_j r_j b_j t$$

- thus

$$\alpha_i\left(t\right) = \frac{-N_i r_i t + \left(w_i C + \sum_{j \neq i} \frac{w_i}{\sum_{k \neq j} w_k} \cdot \left(w_j C - N_j r_j\right)\right)\left(t + d_i\right)}{\sqrt{N_i r_i b_i t + \sum_{j \neq i} \left(\frac{w_i}{\sum_{k \neq j} w_k}\right)^2 N_j r_j b_j \left(t + d_i\right)}}$$

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The GPS scheduler

- The absolute minimum of *ai*(*t*) is

$$\alpha_{i,\min} = \sqrt{\frac{4N_i r_i d_i \left(w_i b_i C + b_i A_i + B_i\right)\cdot\left(w_i C + A_i - N_i r_i\right)}{\left(N_i r_i b_i + B_i\right)^2}}$$

- where

$$A_i = \sum_{j\neq i}\frac{w_i}{\sum_{k\neq j} w_k}\cdot\max\left(0, w_j C - N_j r_j\right);\quad B_i = \sum_{j\neq i}\left(\frac{w_i}{\sum_{k\neq j} w_k}\right)^2 N_j r_j b_j.$$

The GPS scheduler: probability of violation of delay bound

- Thus,

$$\Pr\left(D_i > d_i\right) \approx \exp\left(-\frac{2N_i r_i\left(w_i b_i C + b_i A_i + B_i\right)\cdot\left(w_i C + A_i - N_i r_i\right)}{\left(N_i r_i b_i + B_i\right)^2} d_i\right).$$

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The GPS scheduler: admission control rule

- For the admission control rule, we define

$$N_{i,\min}=\frac{2d_i\left(w_iC\right)^2}{r_i\left(2d_iw_iC-b_i\ln p_i\right)}.\qquad N_{i,\max}=\frac{d_i\left(w_iC+A_i\right)\left(w_ib_iC+b_iA_i+B_i\right)+b_iB_i\ln p_i+\sqrt{d_i\left(w_ib_iC+b_iA_i+B_i\right)^2\left(d_i\left(w_iC+A_i\right)^2+2B_i\ln p_i\right)}}{r_i\left(2d_i\left(w_ib_iC+b_iA_i+B_i\right)-b_i^2\ln p_i\right)}.$$

- Then the following condition is checked

$$\begin{aligned}&If\quad \left(d_i\left(w_iC+A_i\right)^2+2B_i\ln p_i\right)>0\quad then\quad N_i\le\max\left(N_{i,\min},N_{i,\max}\right)\\&else\quad N_i\le N_{i,\min}\end{aligned}$$

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SP versus GPS: an example

- The figure shows a comparison between the admission curves of a Strict Priority and a GPS scheduler
- The GPS scheduler guarantees a minimum bandwidth for all classes
- Thus, service class 2 is not truncated as in the case of the SP scheduler

r1	600000
b1	40000
r2	800000
b2	100000
d1	0.01
d2	0.05
p1	0.000001
p2	0.000001
C	45000000
w1	0.8
w2	0.2