The EDF scheduler

- Given an EDF scheduler with *n* service classes and delay deadline δ_i for service class *i*, the service envelope of class *i* is

$$S_i(t) = \max\left(0, Ct - \sum_{n \neq i} X_n\left(t - \max\left(0, \delta_n - \delta_i\right)\right)\right)$$

- Thus,

$$\alpha_i(t) = -\frac{E\left(X_i(t) - \max\left(0, C\left(t + d_i\right) - \sum_{n \neq i} X_n\left(t + d_i - \max\left(0, \delta_n - \delta_i\right)\right)\right)\right)}{\sqrt{\operatorname{var}\left(X(t) - \max\left(0, C\left(t + d_i\right) - \sum_{n \neq i} X_n\left(t + d_i - \max\left(0, \delta_n - \delta_i\right)\right)\right)\right)}}$$

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The EDF scheduler

- The average value and variance of the service envelope, for linear-variance bounded traffic, are

$$E\left(S_i(t)\right) \approx Ct - \sum_{n \neq i} N_n r_n \left(t - \max\left(0, \delta_n - \delta_i\right)\right)$$
$$\operatorname{var}\left(S_i(t)\right) \approx \sum_{n \neq i} N_n r_n b_n \left(t - \max\left(0, \delta_n - \delta_i\right)\right)$$

- Thus,

$$\alpha_{i,\min} = \frac{-2\left(\left(A_i d_i - E_i - C d_i\right)\left(B_i + N_i r_i b_i\right) - \left(B_i d_i - F_i\right)\left(A_i - C + N_i r_i\right)\right)}{\left(B_i + N_i r_i b_i\right)\sqrt{-\left(B_i d_i - F_i\right) + \dfrac{\left(A_i d_i - E_i - C d_i\right)\left(B_i + N_i r_i b_i\right)}{A_i - C + N_i r_i}}}$$

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$$\alpha_{i,\min} = \frac{-2\left(\left(A_i d_i - E_i - C d_i\right)\left(B_i + N_i r_i b_i\right) - \left(B_i d_i - F_i\right)\left(A_i - C + N_i r_i\right)\right)}{\left(B_i + N_i r_i b_i\right)\sqrt{-\left(B_i d_i - F_i\right) + \frac{\left(A_i d_i - E_i - C d_i\right)\left(B_i + N_i r_i b_i\right)}{A_i - C + N_i r_i}}}$$

- where

$$A_i = \sum_{n \neq i} N_n r_n \qquad E_i = \sum_{n \neq i} N_n r_n \max\left(0, \delta_n - \delta_i\right)$$

$$B_i = \sum_{n \neq i} N_n r_n b_n \qquad F_i = \sum_{n \neq i} N_n r_n b_n \max\left(0, \delta_n - \delta_i\right)$$

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The EDF scheduler

- The probability of violation of the delay bound is

$$\Pr\left(D_i > d_i\right) \le \exp\left(-2\left(N_i r_i d_i\left(b_i\left(C - A_i\right) + B_i\right) + E_i\left(B_i + N_i r_i b_i\right) + F_i\left(C - N_i r_i - A_i\right)\right)\frac{\left(C - N_i r_i - A_i\right)}{\left(B_i + N_i r_i b_i\right)^2}\right)$$

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The EDF scheduler

- The capacity planning formula is

$$C_i \geq \frac{\left(A_i d_i - E_i + \sqrt{\left(E_i + N_i r_i d_i \right)^2 - 2 \ln p_i \left(F_i + N_i r_i b_i d_i \right)} \right)}{2\left(F_i + N_i r_i b_i d_i \right)} \left(B_i + N_i r_i b_i \right) + \frac{1}{2} \left(A_i + N_i r_i \right)$$

- then

$$C = \sup_{1 \leq i \leq n} \left(C_i \right)$$

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The EDF scheduler

- The admission control formula is

$$N_i \le \frac{1}{r_i\left(2\left(b_i\left(Cd_i - A_i d_i + E_i\right) + B_i d_i - F_i\right) - b_i^2 \ln p_i\right)} \times$$

$$\times \left(\begin{array}{l} \left(A_i b_i + B_i - Cb_i\right)\left(A_i d_i - E_i - Cd_i\right) + \\ -2\left(B_i d_i - F_i\right)\left(A_i - C\right) + b_i B_i \ln p_i + \\ + \sqrt{\left(-A_i b_i + B_i + Cb_i\right)^2 \left(\left(A_i d_i - E_i - Cd_i\right)^2 + 2\left(B_i d_i - F_i\right) \ln p_i\right)} \end{array} \right)$$

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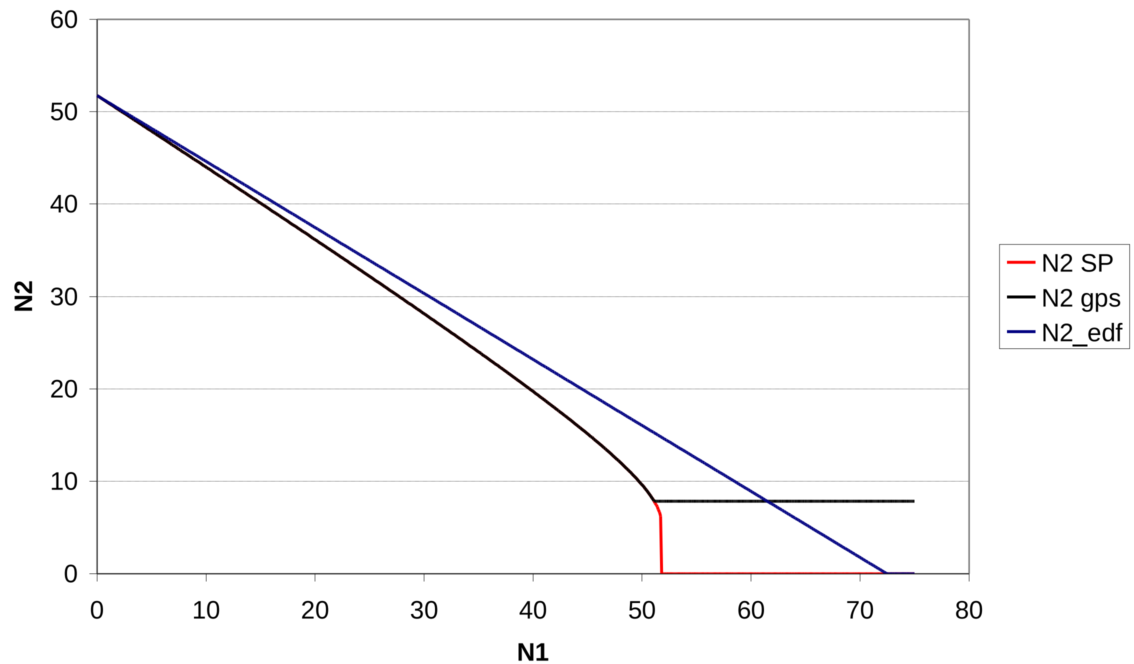
SP versus GPS versus EDF

- The figure plots a comparison among the admission regions of a SP scheduler, a GPS scheduler and a EDF scheduler
- The regiorn are qualitatively different
- The different can change for different sets of parameters

r1	600,000.00	bit/s	r2	800,000.00	bit/s
b1	40,000.00	bit	b2	100,000.00	bit
d1	0.01	s	d2	0.05	s
p1	0.01		p2	0.02	
delta 1	0.10	s	delta2	0.05	s
w1	0.80		w2	0.20	



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