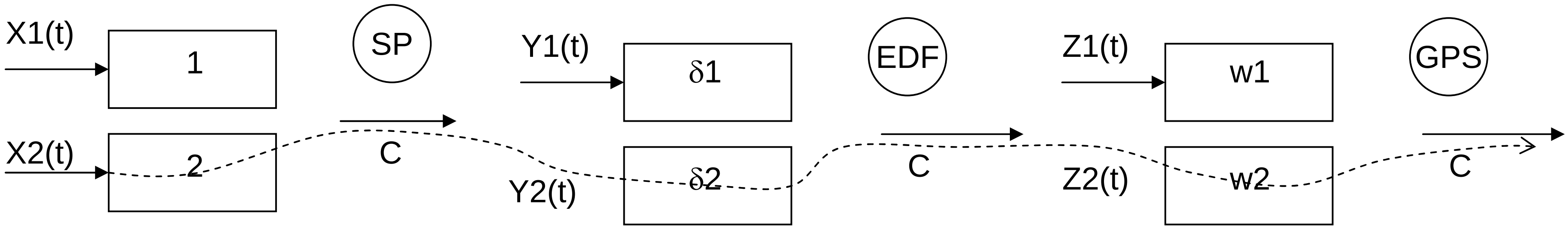
A multihop example



$$X_1(t), X_2(t), Y_1(t), Z_1(t) \equiv \begin{cases} \text{average value} & rt \\ \text{variance} & rbt \end{cases}$$

- Calculate the probability density of the end-to-end delay of the traffic flow X2(t)

In the first scheduler (SP)

$$E\left(S_{X_2}(t)\right) = Ct - E\left(X_1(t)\right) = Ct - rt$$

$$\operatorname{var}\left(S_{X_2}(t)\right) = \operatorname{var}\left(X_1(t)\right) = rbt$$

$$\alpha_{SP}(t) = -\frac{E\left(X_2(t)\right) - E\left(S_{X_2}(t+d)\right)}{\sqrt{\operatorname{var}\left(X_2(t)\right) + \operatorname{var}\left(S_{X_2}(t+d)\right)}} = \frac{C(t+d) - r(2t+d)}{\sqrt{rb(2t+d)}}$$

$$\Pr\left(D_1 > d\right) = \exp\left(-\frac{C(C-2r)}{2rb}d\right) = e^{-k_1 d}; \quad k_1 = \frac{C(C-2r)}{2rb}$$

$$f_{D_1}(t) = k_1 e^{-k_1 t}$$

In the second scheduler (EDF)

$$E\left(Y_2(t)\right)=E\left(X_2(t)\right)=rt$$

$$\mathrm{var}\left(Y_2(t)\right)=\max\left(\mathrm{var}\left(X_2(t)\right),\mathrm{var}\left(S_{X_2}(t)\right)\right)=\max\left(rbt,rbt\right)=rbt$$

$$E\left(S_{Y_2}(t)\right)=Ct-E\left(X_1\left(t-\max\left(0,\delta_1-\delta_2\right)\right)\right)=Ct-r\left(t-\max\left(0,\delta_1-\delta_2\right)\right)$$

$$\mathrm{var}\left(S_{Y_2}(t)\right)=\mathrm{var}\left(X_1\left(t-\max\left(0,\delta_1-\delta_2\right)\right)\right)=rb\left(t-\max\left(0,\delta_1-\delta_2\right)\right)$$

$$\alpha_{EDF}(t)=-\frac{E\left(Y_2(t)\right)-E\left(S_{Y_2}(t+d)\right)}{\sqrt{\mathrm{var}\left(Y_2(t)\right)+\mathrm{var}\left(S_{Y_2}(t+d)\right)}}=\frac{-rt+C(t+d)-r\left(t+d-\max\left(0,\delta_1-\delta_2\right)\right)}{\sqrt{rbt+rb\left(t+d-\max\left(0,\delta_1-\delta_2\right)\right)}}$$

$$\Pr\left(D_1>d\right)=\exp\left(-\frac{C(C-2r)}{2rb}d\right)\exp\left(-\frac{C(C-2r)\max\left(0,\delta_1-\delta_2\right)}{2rb}\right)=$$

$$=ae^{-k_2 d};\quad k_2=\frac{C(C-2r)}{2rb}=k_1;a=e^{-\frac{C(C-2r)\max\left(0,\delta_1-\delta_2\right)}{2rb}}$$

$$f_{D_2}(t)=(1-a)\delta(t)+ak_1e^{-k_1 t}$$

In the third scheduler (GPS)

$$E\left(Z_2(t)\right)=E\left(Y_2(t)\right)=rt$$

$$\text{var}\left(Z_2(t)\right)=\max\left(\text{var}\left(Y_2(t)\right),\text{var}\left(S_{Y_2}(t)\right)\right)=\max\left(rbt,rb\left(t-\max\left(0,\delta_1-\delta_2\right)\right)\right)=rbt$$

$$E\left(S_{Z_2}(t)\right)=w_2Ct+w_1Ct-E\left(Z_1(t)\right)=Ct-rt$$

$$\text{var}\left(S_{Z_2}(t)\right)=\text{var}\left(Z_1(t)\right)=rbt$$

$$\alpha_{GPS}(t)=-\frac{E\left(Z_2(t)\right)-E\left(S_{Z_2}(t+d)\right)}{\sqrt{\text{var}\left(Z_2(t)\right)+\text{var}\left(S_{Z_2}(t+d)\right)}}=\frac{C(t+d)-r(2t+d)}{\sqrt{rb(2t+d)}}=\alpha_{SP}(t)$$

$$\Pr\left(D_3>d\right)=\exp\left(-\frac{C(C-2r)}{2rb}d\right)=e^{-k_1d};\quad k_1=\frac{C(C-2r)}{2rb}$$

$$f_{D_3}(t)=k_1e^{-k_1t}$$

End-to-end delay

$$D_{e2e} = D_1 + D_2 + D_3$$

$$f_{D_{e2e}}(t) = f_{D_1}(t) * f_{D_2}(t) * f_{D_3}(t) = k_1 e^{-k_1 t} * \left((1-a)\delta(t) + a k_1 e^{-k_1 t} \right) * k_1 e^{-k_1 t}$$

$$f_{D_{e2e}}(t) = a \times k_1 e^{-k_1 t} * k_1 e^{-k_1 t} * k_1 e^{-k_1 t} + (1-a)\delta(t) * k_1 e^{-k_1 t} * k_1 e^{-k_1 t}$$

$$f_{D_{e2e}}(t) = \frac{a k_1^3 t^2}{2} e^{-k_1 t} + (1-a) k_1^2 t e^{-k_1 t}$$

In general: convolution of exponential deviates

$$D = \sum_{i=1}^{n} D_i$$

$$f_{D_i}(t) = k_i e^{-k_i t}; \forall i \neq j : k_i \neq k_j$$

$$f_D(t) = \sum_{i=1}^{n} K_i e^{-k_i t}$$

$$K_i = \left\{ \left[\prod_{j=1}^{n} \frac{k_j}{k_j + s} \right] (s + k_i) \right\}_{s=-k_i}$$

In general: convolution of exponential deviates

$$D = \sum_{i=1}^{n} D_i$$

$$f_{D_i}(t) = ke^{-kt}$$

$$f_D(t) = \frac{k^n t^{n-1}}{(n-1)!} e^{-kt}$$