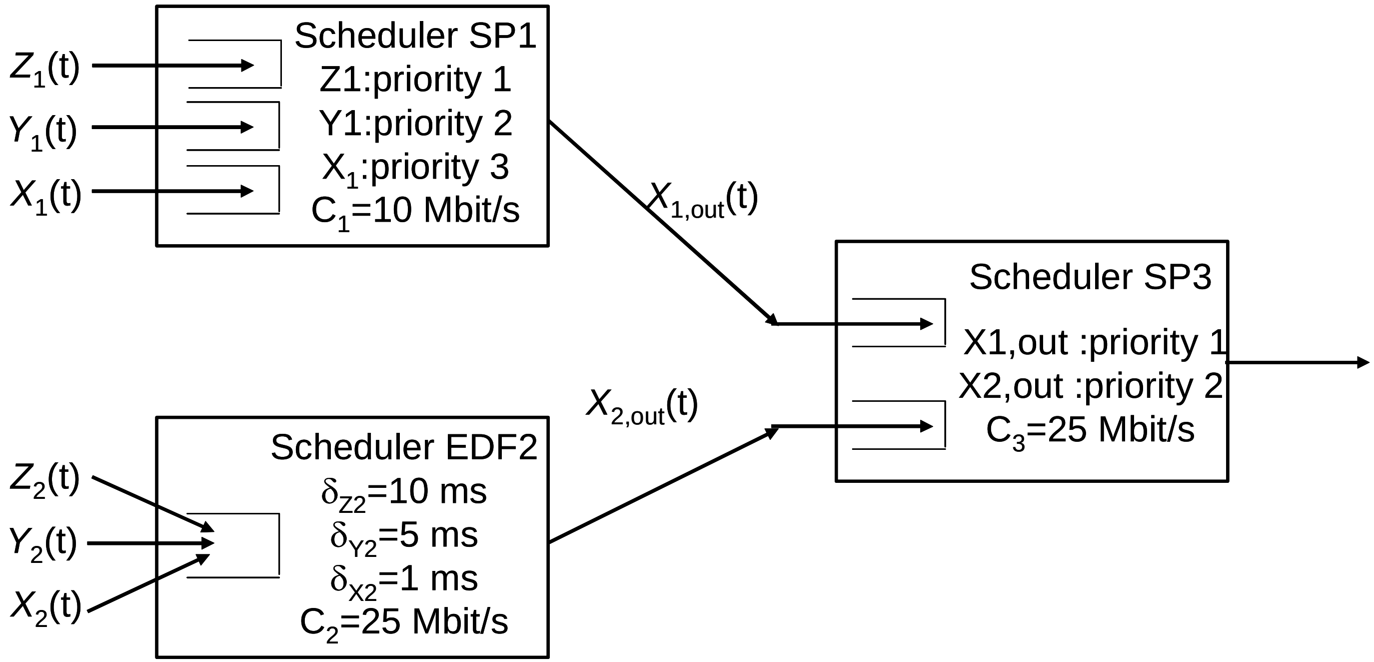
A multinode case



Statement of the problem

- In the figure, X1(t) has a linear variance envelope with parameters r_X1=3Mbit/s and b_X1=100 kbit
- Y1(t) is fGt with parameters m_Y1 = 3.2 Mbit/s, a_Y1 = 5 Mbit, H_Y1=0.99
- Z1(t) has a linear variance envelope with parameters r_Z1=2Mbit/s, b_Z1=150 kbit
- Scheduler 1 is SP; flows X1(t), Y1(t) and Z1(t) are served with priority 3, 2 and 1, respectively
- The capacity of the scheduler output line is 10 Mbit/s
- X1,out(t) is the output of X1(t)
- X2(t) has a linear variance envelope with parameters r_X2= 4 Mbit/s, b_X2= 12.5 kbit
- Y2(t) is fGt with parameters m_Y2 = 2.5 Mbit/s, a_Y2 = 5 Mbit , H_Y2 = 0.85
- Z2(t) has a linear variance envelope with parameters r_Z2= 1 Mbit/s, b_Z2= 50 kbit
- Scheduler 2 is EDF, X2(t) has service deadline δX2=1 ms, Y2(t) has service deadline deadline δY2=5 ms and Z2(t) has service deadline δZ2=10 ms
- The capacity of the scheduler output line is 25 Mbit/s
- ìX2,out(t) is the output of X2(t)
- Scheduler 3 is SP where X1,out(t) is served with priority 1 and X2,out(t) with priority 2
- The capacity of the scheduler output line is 25 Mbit/s
- **Calculate the probability that the delay of flow X2,out(t) exceeds 90 ms in scheduler 3**

Solution (scheduler 1)

- Let us consider scheduler 1; the input flows have average value and variance given by

$$\begin{cases} E\left(Z_1\left(t\right)\right) = r_{Z_1} t = 2\times 10^6 t \text{ bit/s} \\ \operatorname{var}\left(Z_1\left(t\right)\right) = r_{Z_1} b_{Z_1} t = 2\times 10^6 \times 0.15\times 10^6 t \ = 0.3\times 10^6 t \text{ bit}^2\text{/s} \end{cases}$$

$$\begin{cases} E\left(Y_1\left(t\right)\right) = m_{Y_1} t = 3.2\times 10^6 t \text{ bit/s} \\ \operatorname{var}\left(Y_1\left(t\right)\right) = m_{Y_1} a_{Y_1} t^{2H_{Y1}} = 3.2\times 10^6 \times 5\times 10^6 t^{1.98} = 16\times 10^{12} t^{1.98} \text{ bit}^2\text{/s} \end{cases}$$

$$\begin{cases} E\left(X_1\left(t\right)\right) = r_{X_1} t = 3\times 10^6 t \text{ bit/s} \\ \operatorname{var}\left(X_1\left(t\right)\right) = r_{X_1} b_{X_1} t = 0.3\times 10^{12} t \text{ bit}^2\text{/s} \end{cases}$$

Solution (scheduler 1)

- The service envelope for X1(t) is

$$S_{X_1}(t) = Ct - Y_1(t) - Z_1(t)$$

$$E\left(S_{X_1}(t)\right) = Ct - E\left(Y_1(t)\right) - E\left(Z_1(t)\right) = 10\times10^6 t - 3.2\times10^6 t - 2\times10^6 t = 4.8\times10^6 t \text{ bit/s}$$

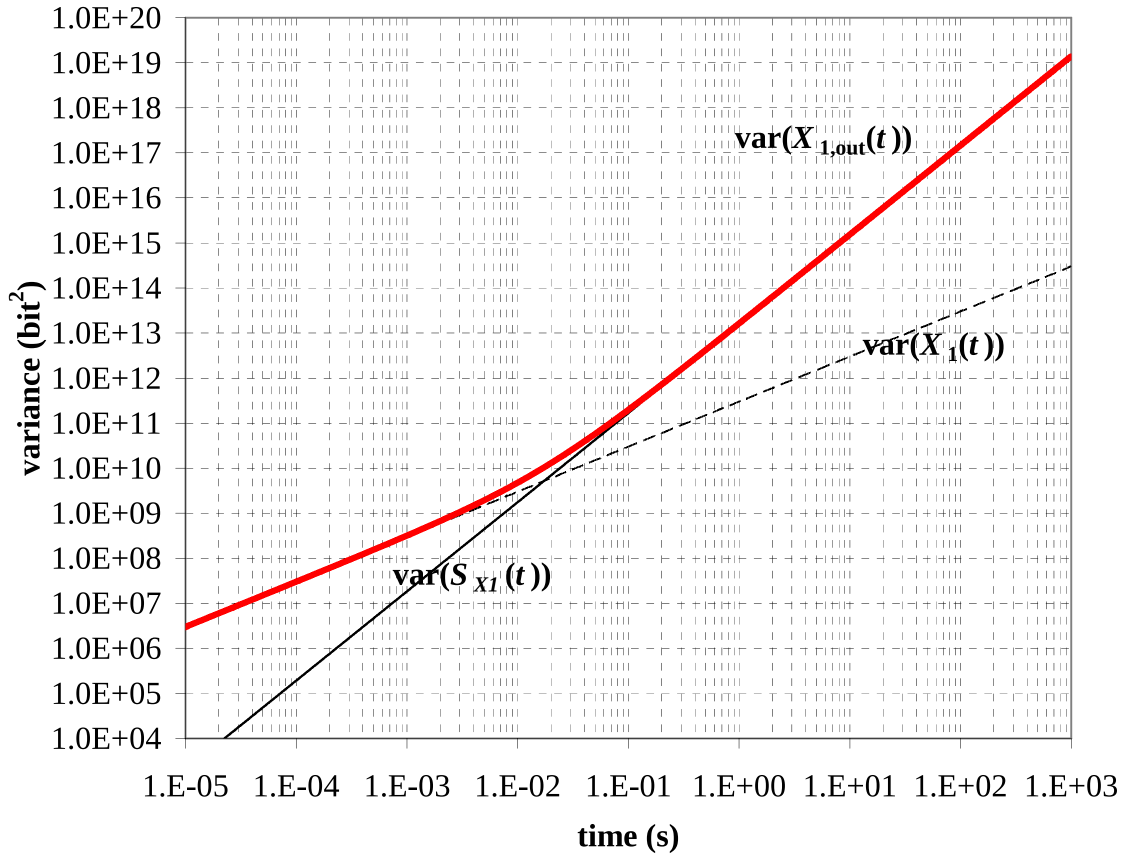
$$\mathrm{var}\left(S_{X_1}(t)\right) = \mathrm{var}\left(Y_1(t)\right) + \mathrm{var}\left(Z_1(t)\right) = 0.3\times10^6 t + 16\times10^{12} t^{1.98} \text{bit}^2\text{/s}$$

- The average value and variance of X1out(t) are

$$E\left(X_{1,\mathrm{out}}(t)\right) = E\left(X_1(t)\right)$$

$$\mathrm{var}\left(X_{1,\mathrm{out}}(t)\right) = \max\left(\mathrm{var}\left(X_1(t)\right), \mathrm{var}\left(S_{X_1}(t)\right)\right)$$

Solution (scheduler 1)



- The figure shows the variance of X1out(t)

Solution (scheduler 2)

- In scheduler 2, the inputs are

$$\begin{cases} E\left(Z_2(t)\right) = r_{Z_2} t = 1\times 10^6 t \text{ bit/s} \\ \text{var}\left(Z_2(t)\right) = r_{Z_2} b_{Z_2} t = 1\times 10^6 \times 0.05\times 10^6 t \ = 0.05\times 10^6 t \text{ bit}^2\text{/s} \end{cases}$$

$$\begin{cases} E\left(Y_2(t)\right) = m_{Y_2} t = 2.5\times 10^6 t \text{ bit/s} \\ \text{var}\left(Y_2(t)\right) = m_{Y_2} a_{Y_2} t^{2H_{Y2}} = 2.5\times 10^6 \times 5\times 10^6 t^{1.7} = 12.5\times 10^{12} t^{1.7} \text{ bit}^2\text{/s} \end{cases}$$

$$\begin{cases} E\left(X_2(t)\right) = r_{X_2} t = 4\times 10^6 t \text{ bit/s} \\ \text{var}\left(X_2(t)\right) = r_{X_2} b_{X_2} t = 4\times 10^6 \times 0.0125\times 10^6 t = 0.05\times 10^{12} t \text{ bit}^2\text{/s} \end{cases}$$

Solution (scheduler 2)

- The service envelope for X2(t) is

$$E\left(S_{X_2}(t)\right) = Ct - m_{Y_2}\left(t - \max\left(\delta_{Y_2} - \delta_{X_2}\right)\right) - r_{Z_2}\left(t - \max\left(\delta_{Z_2} - \delta_{X_2}\right)\right)$$

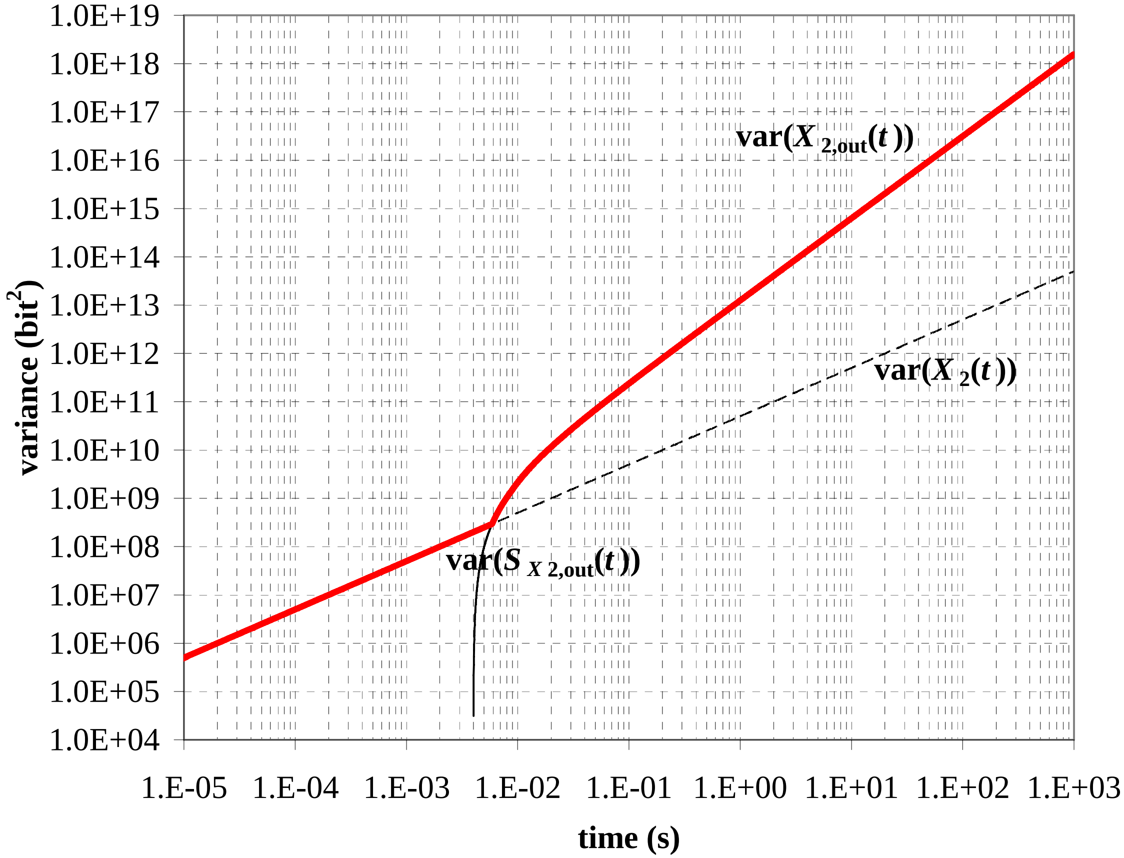
$$\mathrm{var}\left(S_{X_2}(t)\right) = m_{Y_2} a_{Y_2}\left(t - \max\left(\delta_{Y_2} - \delta_{X_2}\right)\right)^{2H_{Y2}} + r_{Z_2} b_{Z_2}\left(t - \max\left(\delta_{Z_2} - \delta_{X_2}\right)\right)$$

- The average value and variance of X1out(t) are

$$E\left(X_{2,\mathrm{out}}(t)\right) = E\left(X_2(t)\right)$$

$$\mathrm{var}\left(X_{2,\mathrm{out}}(t)\right) = \max\left(\mathrm{var}\left(X_2(t)\right), \mathrm{var}\left(S_{X_2}(t)\right)\right)$$

Solution (scheduler 2)



- The figure shows the variance of X2out(t)

Solution (scheduler 3)

- In scheduler 3, X2out(t) has service envelope with the following features

$$S_{X_{2,out}}(t) = Ct - X_{1,out}(t)$$

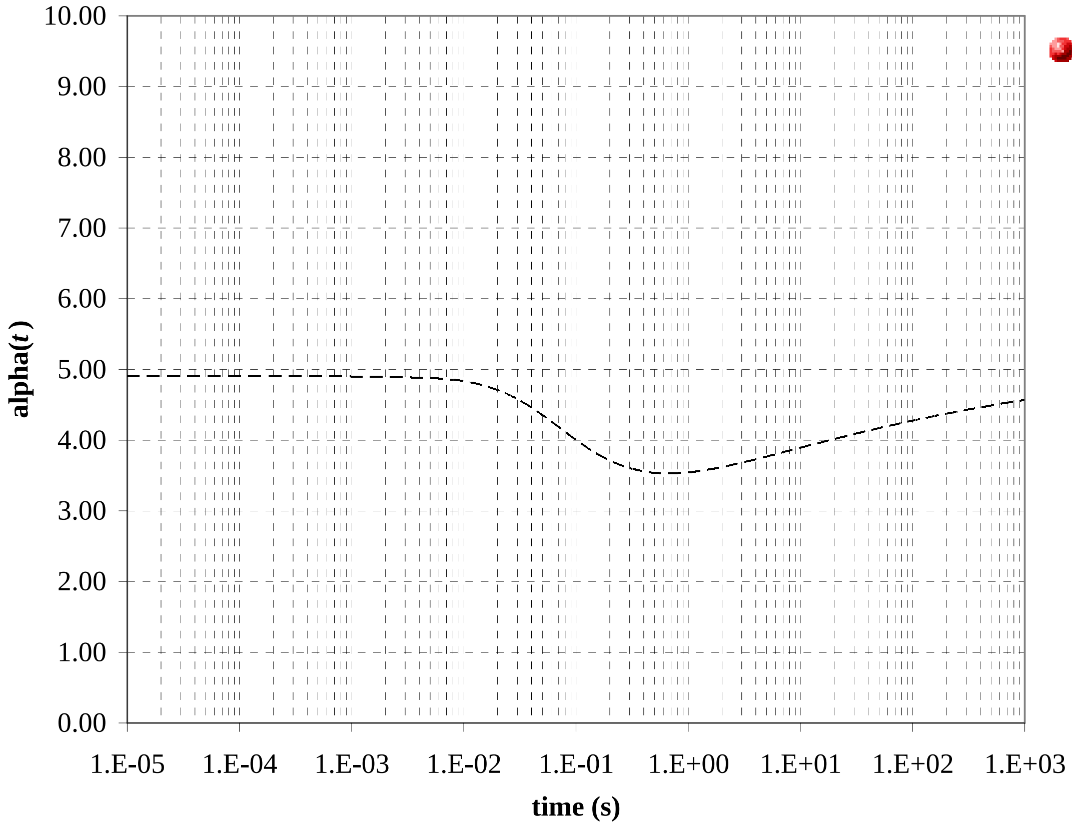
$$E\left(S_{X_{2,out}}(t)\right) = Ct - E\left(X_{1,out}(t)\right) = Ct - E\left(X_1(t)\right)$$

$$\text{var}\left(S_{X_{2,out}}(t)\right) = \text{var}\left(X_{1,out}(t)\right)$$

- The $\alpha(t)$ function is then given by

$$\alpha_{X2,out(t)} = -\frac{E\left(X_{2,out}(t)\right) - E\left(S_{X_{2,out}}(t+d)\right)}{\sqrt{\text{var}\left(X_{2,out}(t)\right) + \text{var}\left(S_{X_{2,out}}(t+d)\right)}}$$

Solution (scheduler 3)

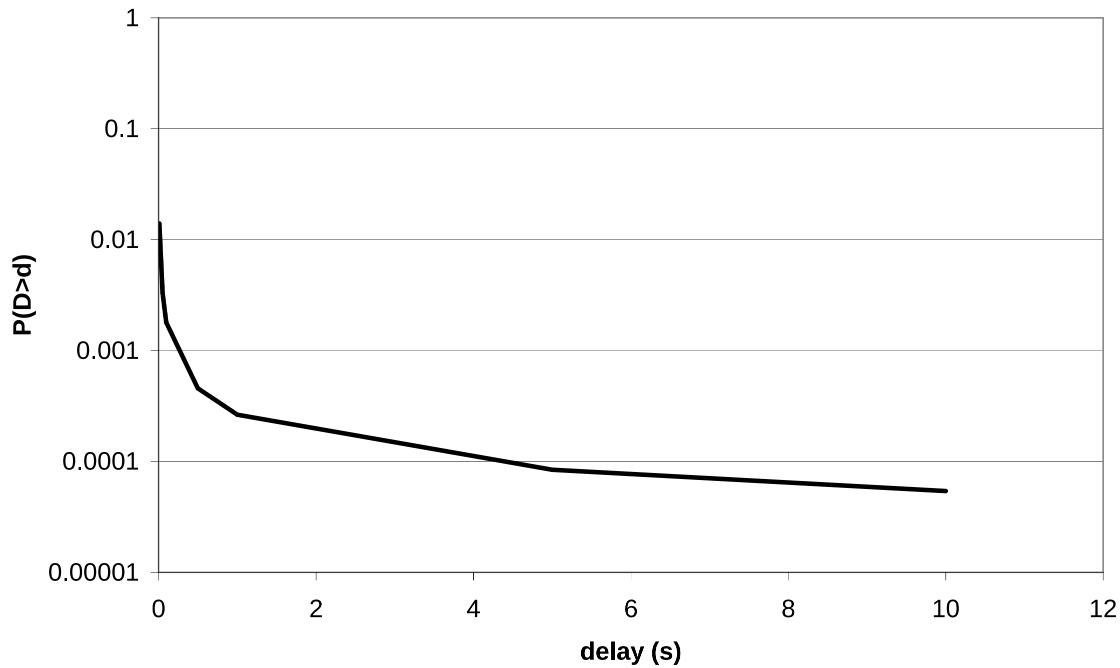


- The minimum value of alpha is 3.53 thus the probability of exceeding the delay threshold is equal to 0.0019

- The figure plots the alpha function

Further considerations

- The procedure can be repeated with different delay thresholds, as shown in the figure
- Remarkably, the delay distribution of X2out in scheduler 3 is fat-tailed
- This is due to the interference form LRD flows

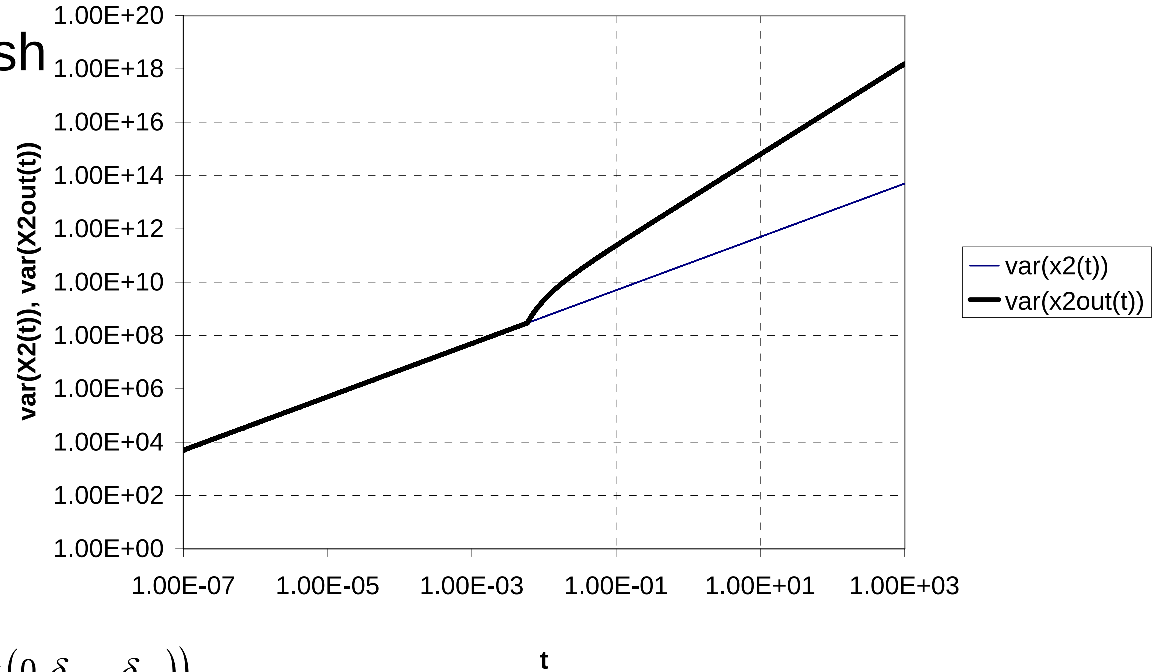


On the variance of X2out(t)

- It is interesting to note that the variance of X2out(t) differs significantly from that of the fresh flow X2(t)
- We recall that

$$\begin{cases} E\left(X_2(t)\right) = 4\times10^6 t \text{ bit} \\ \mathrm{var}\left(X_2(t)\right) = 0.05\times10^{12} t \text{ bit}^2 \end{cases}$$

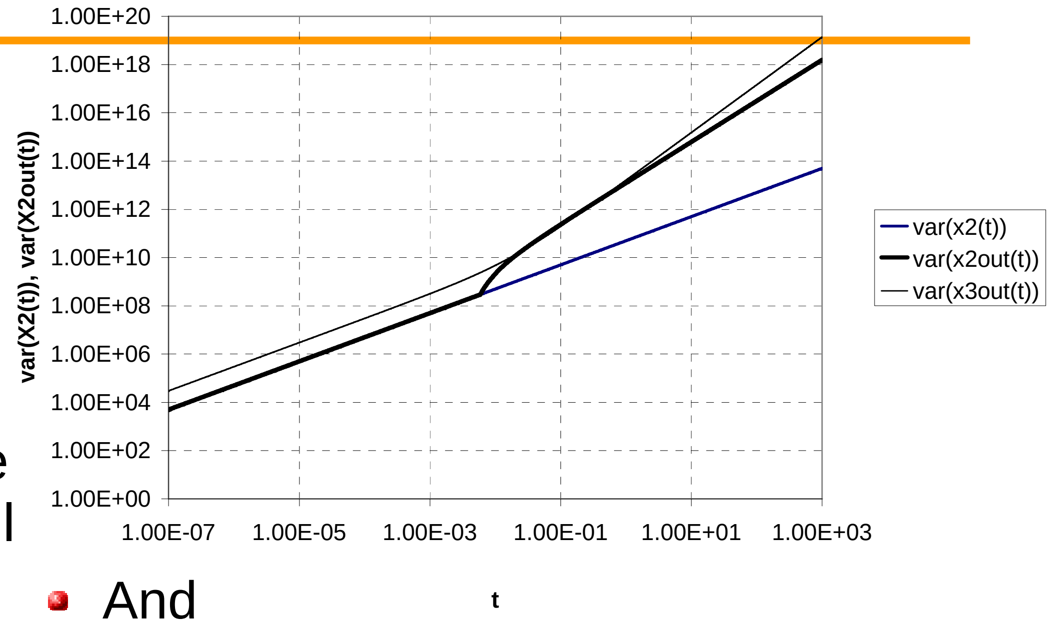
- Then, for $S_{X2}(t)$ we have



$$\begin{cases} E\left(S_{X_2}(t)\right) = Ct - m_{Y_2}\left(t - \max\left(0, \delta_{Y_2} - \delta_{X_2}\right)\right) - r_{Z_2}\left(t - \max\left(0, \delta_{Z_2} - \delta_{X_2}\right)\right) \\ \mathrm{var}\left(S_{X_2}(t)\right) = m_{Y_2} a_{Y_2}\left(t - \max\left(0, \delta_{Y_2} - \delta_{X_2}\right)\right)^{2H_{Y2}} + r_{Z_2} b_{Z_2}\left(t - \max\left(0, \delta_{Z_2} - \delta_{X_2}\right)\right) \end{cases}$$

$$\begin{cases} E\left(S_{X_2}(t)\right) = 25\times10^6 t - 25\times10^6\left(t - 4\times10^{-3}\right) - 1\times10^6\left(t - 9\times10^{-3}\right) \\ \mathrm{var}\left(S_{X_2}(t)\right) = 125\times10^{12}\times\left(t - 4\times10^{-3}\right)^{1.7} + 50\times10^9\left(t - 9\times10^{-3}\right) \end{cases}$$

On the variance of X2out(t)



- Thus,

$$\mathrm{var}\left(X_{2,\mathrm{out}}(t)\right)=\max\left(\mathrm{var}\left(X_2(t)\right),\mathrm{var}\left(S_{X_2}(t)\right)\right)=$$

$$=\max\left(0.05\times10^{12}t,125\times10^{12}\times\left(t-4\times10^{-3}\right)^{1.7}+50\times10^{9}\left(t-9\times10^{-3}\right)\right)$$

- Then, $X_{2out}(t)$ is offered to the third Strict Priority scheduler, and we want to calculate the variance of the output flow of this scheduler (we call it $X_{3out}(t)$)
- $\mathrm{var}(X_{3out}(t))=\max(\mathrm{var}(X_{2out}(t)), S(X_{2out}(t)))$:

$$\mathrm{var}\left(S_{X_{2,out}}(t)\right)=\mathrm{var}(X_{1,out}(t))$$

- where

$$\mathrm{var}\left(X_{1,out}(t)\right)=\max\left(\mathrm{var}\left(X_1(t)\right),\mathrm{var}\left(S_{1,out}(t)\right)\right)$$

- And

$$\mathrm{var}\left(S_{X1,out}(t)\right)=\mathrm{var}\left(Z_1(t)\right)+\mathrm{var}\left(Y_1(t)\right)=$$

$$=2\times10^6\times150\times10^3t+3.2\times10^6\times5\times10^6\times t^{1.98}=$$

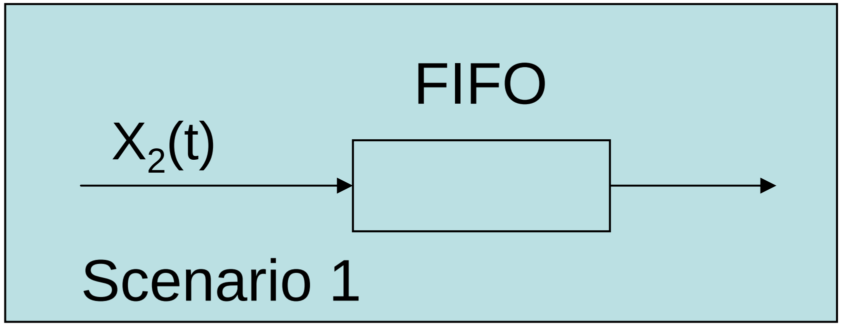
$$=300\times10^9t+16\times10^{12}\times t^{1.98}$$

- Thus

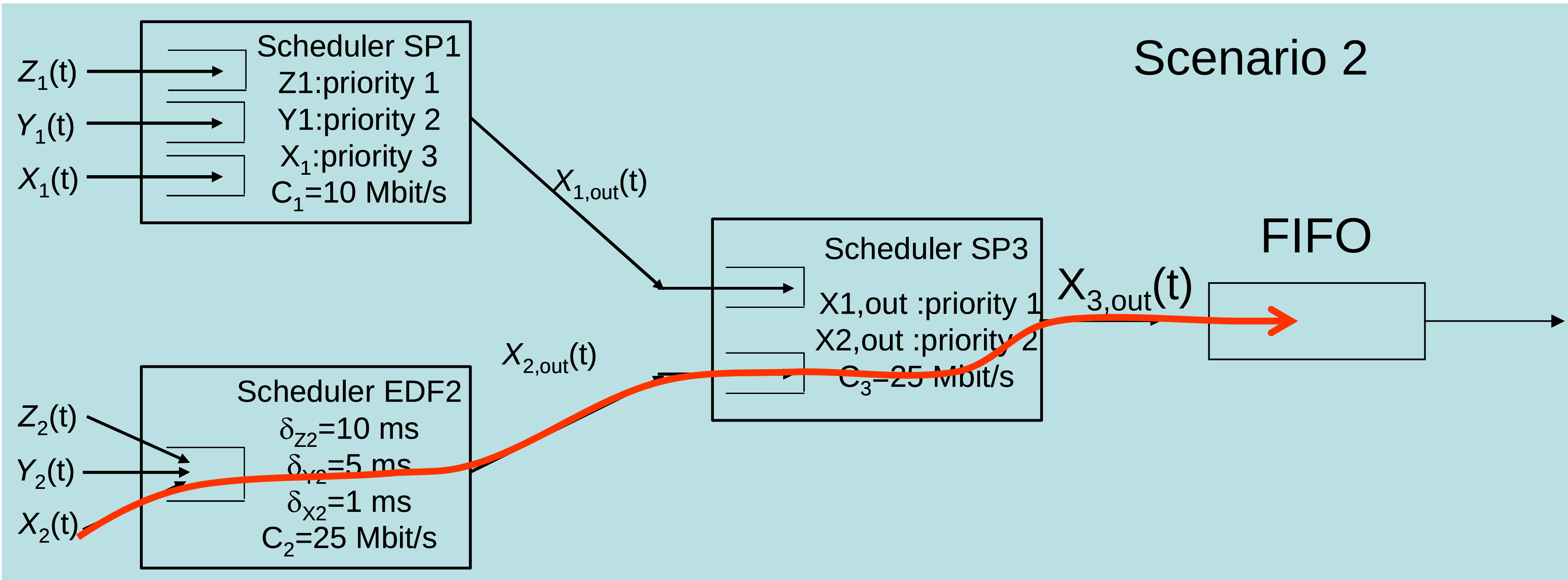
$$\mathrm{var}\left(X_{3,out}(t)\right)=$$

$$\max\left(300\times10^9t,\max\left(300\times10^9t,300\times10^9t+16\times10^{12}\times t^{1.98}\right)\right)$$

Analysis of the behavior of X3out(t)



- Now, we consider two scenarios
  - Scenario 1: X2(t) is directly offered to a FIFO scheduler
  - Scenario 2: X2(t) is offered to the same FIFO scheduler after having crossed the network (i.e., we offer X3out(t))



Analysis of the behavior of X3out(t)

- Now, the calculation is simple, as it is sufficient to calculate the alpha function in both cases:

$$\alpha_{X_2}(t) = -\frac{m_{X_2}t - C(t+d)}{\sqrt{\mathrm{var}(X_2(t))}}; \alpha_{X_{3,out}}(t) = -\frac{m_{X_2}t - C(t+d)}{\sqrt{\mathrm{var}(X_{3,out}(t))}};$$

- By repeating the calculation for different values of d, we obtain the delay curve in both cases

Analysis of the behavior of X3out(t)

- The difference between the queueing behavior of X2(t) and X3out(t) is impressive
- X2(t) acquires a heavy LRD when it crosses the network
- Actually, it interferes with a flow with *H*=0.99, an extremely high value of hurst parameter
- X2(t) seems to acquire a LRD behavior
- This is a known fact: LRD tends to spread in the Internet
- However, the methods described in this courses are the first allowing us to calculate analytically this kind of behavior