Fading channels

Fading channels

- Fading channels can be included in the present framework of network calculus, as in some cases they can be modeled by a Markov chain
- For example, H. S. Wang and N. Moayeri (1995) propose a discrete-time finite-state Markov model of the Rayleigh fading channel
- In this model, the SINR range is partitioned into a finite number of intervals and the transition probabilities are obtained from the level crossing rate associated to the fading process
- Moreover, Q. Zhang and S. Kassam (1999) propose a technique for the of optimal choice of the SINR thresholds between states
- This method was later extended to include the Nakagami-m fading case by Qingwen Liu, Shengli Zhou and Giannakis (2005), and to the Rician fading case by Pimentel, Falk, and Lisboa (2004)
- The same authros also discuss the case when Adaptive Modulation and Coding (AMC) is used to match transmission parameters to time-varying channel conditions

Fading channels with AMC

- In this case, the SINR range is naturally partitioned so that each state corresponds to a different Modulation and Coding Scheme (MCS) and the SINR thresholds coincide to the thresholds used by the MCS decision algorithm
- Assuming that N MCSs are available, the decision algorithm uses Channel State Information (CSI) to choose the MCS with the highest spectral efficiency capable of satisfying the BER target requirement
- The main assumptions to use this model are:
  - the channel is frequency flat;
  - the channel coherence time is longer than the frame duration;
  - perfect CSI is available;
  - pathloss and shadowing do not change over time.

Fading channels with AMC

- Given the above assumptions, the radio channel behavior can be captured frame by frame by a single parameter, γ, the SINR at the receiver, which can be modeled as $\gamma=\gamma_0\varepsilon$, where γ_0 is the average SINR and depends only on pathloss and shadowing
- Multipath fading is modeled by the random ε variable, which, in the Rayleigh channel, is a negative exponential random variable with unitary mean
- Sorting the N available MCSs by increasing spectral efficiency, the entire SINR range is partitioned in $N + 1$ consecutive nonoverlapping intervals with boundary points γ_n, with $1 \leq n \leq N$, such that, if $\gamma_n \leq \gamma < \gamma_{n+1}$, then the n-th MCS is the most efficient one capable of maintaining the required BER
- In case $\gamma < \gamma_1$, no such MCS exists

Fading channels with AMC

- We do not discuss the optimal choice of the boundary points, which is the design objective of the AMC selector
- We assume that γ_n is the SINR at which MCS *n* yields exactly the target BER
- In the model, each SINR interval corresponds to a state in a continuous-time FSMC
- The probability of the state *n* is

$$p_n = \int_{\gamma_n}^{\gamma_{n+1}} p_\gamma(\gamma) d\gamma = \exp\left(-\frac{\gamma_{n+1}}{\overline{\gamma}}\right) - \exp\left(-\frac{\gamma_n}{\overline{\gamma}}\right)$$

- Where $p_\gamma(\gamma)$ is the SINR probability density function

Fading channels with AMC

- We assume that transitions happen only between adjacent states, then the matrix of the transition rates, Q, is a band matrix
- The elements on the main diagonal are computed so that each column sums up to zero
- The elements above and below the main diagonal are

$$Q_{n,n+1} = \frac{N_{n+1}}{p_n}, \qquad n = 0, \ldots, N-1$$

$$Q_{n,n-1} = \frac{N_n}{p_n}, \qquad n = 1, \ldots, N$$

- where N_n is the level crossing rate at the boundary point γ_n, which can be estimated as

$$N_n = \sqrt{2\pi \frac{\gamma_n}{\overline{\gamma}}} f_d \exp\left(-\frac{\gamma_n}{\overline{\gamma}}\right)$$

- Where f_d is the doppler spread

Fading channels with AMC

- Given the Markov chain of the fading channel, it is possible to calculate a linear upper bound of the variance of the channel’s capacity
- This is done with a specific theorem (Giacomazzi 2009*)

$$\operatorname{var}(X(t)) \leqslant \left(2\sum_{i=1}^{M} p_i r_i \sum_{j=1}^{M} r_j \sum_{k=1}^{M-1} \frac{\gamma_{j,i,k}}{\omega_k}\right) t.$$

- Where
  - p_i is the probability of occurrence of state *i*
  - r_i is the rate in state *i*
  - ω_k is the *k*th eigenvalue of the markov chain
  - γ_{ijk} is a real constant

- * P. Giacomazzi, Closed-form analysis of end-to-end network delay with Markov-modulated Poisson and fluid traffic”, Elsevier Computer Communications, Volume 32, Issue 4, 4 March 2009, Pages 640-648

Fading channels with AMC

- Now, we consider a WiMAX fading channel with the following features
- The selected modulations and the associated rates and thresholds are shown in table 1
- The frame duration is 2 ms
- The thresholds γ_n are obtained considering a target BER of 10^{-4}
- The fading channel is characterized by its ρ and β parameters in such a way that

$$E\left(S(t)\right) = \rho t$$

$$\mathrm{var}\left(S(t)\right) \le \rho\beta t$$

TABLE I
MODULATION AND CODING SCHEMES

n	MCS	rate (kbit/s)	γ_n
0	–	0	–
1	QPSK 1/2 (2x rep.)	12	-0.06 dB
2	QPSK 1/2	24	3.22 dB
3	QPSK 3/4	36	5.64 dB
4	16QAM 1/2	48	8.42 dB
5	16QAM 3/4	72	11.91 dB
6	64QAM 1/2	72	12.37 dB
7	64QAM 2/3	96	15.25 dB
8	64QAM 3/4	108	17.11 dB

$$\text{Average SNR} = 5\text{ dB: }\begin{cases} E\left(S(t)\right) = 20.240\times10^3 t \text{ bit} \\ \mathrm{var}\left(S(t)\right) = \dfrac{587.36\times10^6}{f_d} t \text{ bit}^2 \end{cases}$$

$$\text{Average SNR} = 10\text{ dB: }\begin{cases} E\left(S(t)\right) = 40.9\times10^3 t \text{ bit} \\ \mathrm{var}\left(S(t)\right) = \dfrac{1222\times10^6}{f_d} t \text{ bit}^2 \end{cases}$$

f_d : measured in Hz

with central frequency 2.5 GHz and user speed 2 km/h: $f_d = 4.6$ Hz

with central frequency 2.5 GHz and user speed 45 km/h: $f_d = 104$ Hz

with central frequency 5 GHz and user speed 2 km/h: $f_d = 10.7$ Hz

with central frequency 5 GHz and user speed 45 km/h: $f_d = 241$ Hz

Fading channels with AMC

- Thus, we have the following result:

Average SNR	Central frequency	User speed	ρ (bit/s)	β (bit)
5 dB	2.5 GHz	2 km/h	20.240×10^3	6.3×10^3
5 dB	2.5 GHz	45 km/h	20.240×10^3	0.27×10^3
5 dB	5 GHz	2 km/h	20.240×10^3	2.71×10^3
5 dB	5 GHz	45 km/h	20.240×10^3	0.12×10^3
10 dB	2.5 GHz	2 km/h	40.9×10^3	6.49×10^3
10 dB	2.5 GHz	45 km/h	40.9×10^3	0.29×10^3
10 dB	5 GHz	2 km/h	40.9×10^3	2.79×10^3
10 dB	5 GHz	45 km/h	40.9×10^3	0.124×10^3

Fading channels with AMC

- Note that 96 channels are available
- They are divided in 24 sets of 4 channels
- The fading process of different sets can be assumed to be approximately independent

Fading channels with AMC

- How to deal with a fading channel?
- The channel's capacity is not constant any longer
- Thus, C=C(t) rather than having a constant capacity C
- We will examine at first the FIFO scheduler, for which

$$\alpha(t) = -\frac{E\left(X(t)-S(t+d)\right)}{\sqrt{\operatorname{var}\left(X(t)-S(t+d)\right)}}$$

- Where, for a fading channel

$$E\left(S(t)\right) = \rho t$$

$$\operatorname{var}\left(S(t)\right) \leq \rho\beta t$$

Fading channels with AMC

- Thus, for a generic input traffic flow,

$$\alpha(t) = -\frac{E(X(t)) - \rho(t+d)}{\sqrt{\mathrm{var}(X(t)) + \rho\beta(t+d)}}$$

- And for a linear-bounded variance input flow:

$$\alpha(t) = -\frac{rt - \rho(t+d)}{\sqrt{rbt + \rho\beta(t+d)}}$$

- The result is

$$\alpha_{\min} = 2\frac{\sqrt{r\rho}(b+\beta)}{\sqrt{(rb+\rho\beta)\frac{b+\beta}{\rho - r}}}\sqrt{d}$$

Fading channels with AMC

- Finally,

$$\Pr(D > d) \approx \exp\left(-2\frac{r\rho(\rho - r)(b+\beta)}{(rb+\beta\rho)^2}d\right)$$

- Now, we calculate the delay curve of a VoIP flow over a fading WiMAX channel
- We assume a 10 dB average SIR, central frequelcy of 2.5 GHz and spped of movement equal to 2 km/h and 45 km/h
- We aggregate two channels, with a total average rate of 81,800 bit/s
- The result is shown in the next slide

Fading channels with AMC