## Sample exam

| COGNOME |  |
| :--- | :--- |
| NOME |  |
| MATRICOLA |  |

## Exercise 1.

In the network shown in the Figure, $N_{1}$ VoIP flows are multiplexed by a FIFO scheduler with transmission capacity equal to $C_{1}$. These flows are then multiplexed by another FIFO scheduler with other $N_{2}$ flussi VoIP; the capacity of the second FIFO scheduler is $C_{2}$.


The VoIP codec is the G. 726 with Voice Activity Detection (VAD). The parameters are:

- $\lambda\left(\mathrm{s}^{-1}\right)$ : transition rate from OFF state (silence) to ON state (voice);
- $\mu\left(\mathrm{s}^{-1}\right)$ : transition rate from ON state to OFF state;
- $\quad P(\mathrm{bit} / \mathrm{s})$ : coded speed, in the ON state (note that this is the application layer speed, it does not take into account protocol overheads);
- $d_{f}(\mathrm{~s})$ : frame duration;
- $d_{a}(\mathrm{~s})$ : algoritmic delay (only in the coding phase);
- $d_{p}(\mathrm{~s})$ : playout buffer depth.

The protocol stack is RTP/UDP/IP/PPP. The total protocol overhead per packet is $L$ (bit).

Calculate the probability density of the queuing delay in the FIFO buffer 1 , and its average value $D_{1 .} D_{1}$ does not include coding delay and the transmission time of VoIP packets.

## Soluzione.

Length of VoIP packets:

$$
L_{\text {VoIP }}=P d_{f}+L \text { (bit). }
$$

Physical-layer peak transmission speed:

$$
P_{\text {phy }}=\frac{L_{\text {VoIP }}}{d_{f}}=\frac{P d_{f}+L}{d_{f}}(\mathrm{bit} / \mathrm{s}) .
$$

Average transmission speed of one VoIP flow, at the physical layer:

$$
r=\frac{\lambda}{\lambda+\mu} P_{p h y}=\frac{\lambda}{\lambda+\mu} \frac{P d_{f}+L}{d_{f}}(\mathrm{bit} / \mathrm{s}) .
$$

Burst lebgth of one VoIP flow:

$$
b=2 \frac{\mu}{(\lambda+\mu)^{2}} P_{p h y}=2 \frac{\mu}{(\lambda+\mu)^{2}} \frac{P d_{f}+L}{d_{f}} \text { (bit). }
$$

For $N_{1}$ flows:

$$
\begin{gathered}
R_{1}=N_{1} \frac{\lambda}{\lambda+\mu} \frac{P d_{f}+L}{d_{f}} . \\
B_{1}=N_{1} 2 \frac{\mu}{(\lambda+\mu)^{2}} \frac{P d_{f}+L}{d_{f}} .
\end{gathered}
$$

Thus, the probability density of delay in the first scheduler is:

$$
f_{D_{1}}(t)=2 \frac{C_{1}\left(C_{1}-R_{1}\right)}{R_{1} B_{1}} \exp \left(-2 \frac{C_{1}\left(C_{1}-R_{1}\right)}{R_{1} B_{1}} t \stackrel{)}{\dot{\varphi}} .\right.
$$

And its average value is:

$$
E\left(D_{1}\right)=\frac{R_{1} B_{1}}{2 C_{1}\left(C_{1}-R_{1}\right)} .
$$

Now, calculate the probability density and average value of delay in the second scheduler.

## Solution.

The available service in the first scheduler is $C_{1} t$, thus, the variance of the traffic at the output of the first scheduler is equal to the variance of traffic at its input. Thus, at the input of scheduler 2:

$$
\begin{gathered}
R_{2}=\left(N_{1}+N_{2}\right) \frac{\lambda}{\lambda+\mu} \frac{P d_{f}+L}{d_{f}} . \\
B_{2}=\left(N_{1}+N_{2}\right) 2 \frac{\mu}{(\lambda+\mu)^{2}} \frac{P d_{f}+L}{d_{f}}
\end{gathered}
$$

Thus:

$$
\begin{gathered}
f_{D_{2}}(t)=2 \frac{C_{2}\left(C_{2}-R_{2}\right)}{R_{2} B_{2}} \exp \left(-2 \frac{C_{2}\left(C_{2}-R_{2}\right)}{R_{2} B_{2}} t \frac{)}{\dot{)}}\right. \\
E\left(D_{2}\right)=\frac{R_{2} B_{2}}{2 C_{2}\left(C_{2}-R_{2}\right)}
\end{gathered}
$$

Now, calculate the end-to-end average delay of traffic, including coding/decoding and transmission delay
Solution.

$$
E\left(D_{1, \text { tot }}\right)=d_{f}+d_{a}+E\left(D_{1}\right)+\frac{L_{\text {VoIP }}}{C_{1}}+E\left(D_{2}\right)+\frac{L_{\text {VoIP }}}{C_{2}}+d_{p}
$$

Si calculate the probability density of the delay $D_{1}+D_{2}$.
Soluzione.

$$
\begin{aligned}
& f_{D_{1}}(t)=2 \frac{C_{1}\left(C_{1}-R_{1}\right)}{R_{1} B_{1}} \exp \left(-2 \frac{C_{1}\left(C_{1}-R_{1}\right)}{R_{1} B_{1}} t \stackrel{-}{)}=k_{1} e^{-k_{1} t}\right. \\
& k_{1}=2 \frac{C_{1}\left(C_{1}-R_{1}\right)}{R_{1} B_{1}} \\
& f_{D_{2}}(t)=2 \frac{C_{2}\left(C_{2}-R_{2}\right)}{R_{2} B_{2}} \exp \left(-2 \frac{C_{2}\left(C_{2}-R_{2}\right)}{R_{2} B_{2}} t\right)=k_{2} e^{-k_{2} t} \\
& k_{2}=2 \frac{C_{2}\left(C_{2}-R_{2}\right)}{R_{2} B_{2}}
\end{aligned}
$$

thus:

$$
f_{D_{1}+D_{2}}(t)=\frac{k_{1} k_{2}}{-k_{1}+k_{2}} e^{-k_{1} t}+\frac{k_{1} k_{2}}{k_{1}-k_{2}} e^{-k_{2} t}
$$

## Exercise 2.

Let us consider a series of two schedulers, with 5 service classes for each scheduler. The first scheduler is Strict Priority, and it serves the traffic flows X1, X2, X3, X4, X5, with priority 1, 2, 3, 4 and 5, respectively. Each flow has parameters $m$ (average rate), $b$ (burst length) and $H$ (Hurst parameter). The traffic flow X5, at the output of the first scheduler, is named Y5 and it is offered to a second scheduler, GPS, that serves also the fresh traffic flows Y1, Y2, Y3 e Y4. The weights of the traffic flows Y1, Y2, Y3, Y4 and Y5 are w1, w2, w3, w4 e w5, respectively. The capacity of schedulers 1 and 2 is C1 and C2, respectively. Calculate the probability that the delay of the flow Y 5 in the second scheduler is larger than d.

Parameters:

- $\mathrm{mx} 1=1 \mathrm{x} 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{bx} 1=1 \mathrm{x} 10^{6}$ bit, $\mathrm{Hx} 1=0.95$;
- $\mathrm{mx} 2=2 \times 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{bx} 2=2 \times 10^{6} \mathrm{bit}, \mathrm{Hx} 2=0.92$;
- $\mathrm{mx} 3=3 \times 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{bx} 3=3 \times 10^{6} \mathrm{bit}, \mathrm{Hx} 3=0.89$;
- $\mathrm{mx} 4=4 \times 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{bx} 4=4 \times 10^{6}$ bit, $\mathrm{Hx} 4=0.87$;
- $\mathrm{mx} 5=5 \times 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{bx} 5=5 \times 10^{6}$ bit, $\mathrm{Hx} 5=0.85$;
- my1 $=1 \times 10^{6}$ bit/s, by $1=1 \times 10^{6}$ bit, Hy1 $=0.96$;
- $\mathrm{my} 2=2 \times 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{by} 2=2 \times 10^{6} \mathrm{bit}, \mathrm{Hy} 2=0.94$;
- $\mathrm{my} 3=3 \times 10^{6} \mathrm{bit} / \mathrm{s}, \mathrm{by} 3=3 \times 10^{6}$ bit, Hy3 $=0.92$;
- my $4=4 \times 10^{6}$ bit/s, by $4=4 \times 10^{6}$ bit, Hy $4=0.90$;
- $w 1=1 / 15, \mathrm{w} 2=2 / 15, \mathrm{w} 3=3 / 15, \mathrm{w} 4=4 / 15$, $\mathrm{w} 5=5 / 15$;
- $\mathrm{C} 2=45 \mathrm{Mbit} / \mathrm{s}$
- $\mathrm{d}=0.05 \mathrm{~s}$


## Solution

$$
\begin{array}{clll}
E\left(X_{1}(t)\right)=m_{X_{1}} t & \operatorname{var}\left(X_{1}(t)\right)=m_{X_{1}} b_{X_{1}} t^{2 H_{X_{1}}} & E\left(Y_{1}(t)\right)=m_{Y_{1}} t & \operatorname{var}\left(Y_{1}(t)\right)=m_{Y_{1}} b_{Y_{1}} t^{2 H_{Y_{1}}} \\
E\left(X_{2}(t)\right)=m_{X_{2}} t & \operatorname{var}\left(X_{2}(t)\right)=m_{X_{2}} b_{X_{2}} t^{2 H_{X_{2}}} & E\left(Y_{2}(t)\right)=m_{Y_{2}} t & \operatorname{var}\left(Y_{2}(t)\right)=m_{Y_{2}} b_{Y_{2}} t^{2 H_{Y_{2}}} \\
E\left(X_{3}(t)\right)=m_{X_{3}} t & \operatorname{var}\left(X_{3}(t)\right)=m_{X_{3}} b_{X_{3}} t^{2 H_{X_{3}}} & E\left(Y_{3}(t)\right)=m_{Y_{3}} t & \operatorname{var}\left(Y_{3}(t)\right)=m_{Y_{3}} b_{Y_{3}} t^{2 H_{Y_{3}}} \\
E\left(X_{4}(t)\right)=m_{X_{4}} t & \operatorname{var}\left(X_{4}(t)\right)=m_{X_{4}} b_{X_{4}} t^{2 H_{X_{4}}} & E\left(Y_{4}(t)\right)=m_{Y_{4}} t & \operatorname{var}\left(Y_{4}(t)\right)=m_{Y_{4}} b_{Y_{4}} t^{2 H_{Y_{4}}} \\
E\left(X_{5}(t)\right)=m_{X_{5}} t & \operatorname{var}\left(X_{5}(t)\right)=m_{X_{5}} b_{X_{5}} t^{2 H_{X_{5}}} & \operatorname{var}\left(S_{X_{5}}(t)\right)=\operatorname{var}\left(X_{1}(t)\right)+\operatorname{var}\left(X_{2}(t)\right)+\operatorname{var}\left(X_{3}(t)\right)+\operatorname{var}\left(X_{4}(t)\right) \\
\operatorname{var}\left(Y_{5}(t)\right)=\max \left(\operatorname{var}\left(X_{5}(t)\right), \operatorname{var}\left(S_{X_{5}}(t)\right)\right)=E\left(X_{5}(t)\right) \\
E\left(S_{Y_{5}}(t)\right)=w_{5} C_{2} t+\sum_{j=1}^{4} \frac{w_{5}}{\sum_{k=1 L} w_{5, k \neq j}\left(w_{j} C_{2} t-E\left(X_{j}(t)\right)\right)}
\end{array}
$$

$$
\begin{aligned}
& \alpha(t)=-\frac{E\left(Y_{5}(t)\right)-E\left(S_{Y_{5}}(t+d)\right)}{\sqrt{\operatorname{var}\left(Y_{5}(t)\right)+\operatorname{var}\left(S_{Y_{5}}(t+d)\right)}}
\end{aligned}
$$

Numerically

$p=\exp \left(-3.17^{2} / 2\right)$.

## Exercise 3.

Explain how applications can be mapped onto PHBs in a Diffserv network. Which kinds of PHBs are properly selected for each application?

